Nuclear Electromagnetic Current in the Relativistic Approach with the Momentum-Dependent Selfenergies

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We define the new description of the electromagnetic current to hold the current conservation in the momentum-dependent Dirac fields from the Ward Takahashi identity. To describe the momentum-dependence we solve the relativistic Hartree-Fock approximation by using the one-pion exchange. In addition we discuss on contribution from the one-pion exchange current and the core polarization. It is shown that this current can reduce the convection current in the isovector case, whose value has been too big due to the small effective mass in the usual relativistic Hartree approximation.

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I. INTRODUCTION

The past decades have seen many successes in the relativistic treatment of the nuclear many-body problem. The relativistic framework has big advantages in several aspects [1]: a useful Dirac phenomenology for the description of nucleon-nucleus scattering [2, 3], the natural incorporation of the spin-orbit force [1] and the saturation properties of nuclear matter in the microscopic treatment with the Dirac Brueckner Hartree-Fock (DBHF) approach [4].

These results conclude that there are large attractive scalar and repulsive vector-fields, and that the nucleon effective mass is very small in the medium. However this small effective mass leads to small Fermi velocity, which makes some troubles in the nuclear properties: too big magnetic-moment [5] and too big excitation energy of the isoscalar giant quadrupole resonance (ISGQR) state [6]. As for the isoscalar magnetic-moment, this enhancement is cancelled by the Ring-Diagram contribution [7]; this relation is completely realized by the gauge invariance [8]. As for the isovector one, however, this contribution does not plays a significant role because the symmetry force is not efficiently large.

In this subject most of people believed that the momentum dependence of the Dirac fields is negligible in the low energy region, particularly below the Fermi level. A momentum dependence

of the Schrödinger equivalent potential automatically emerges as a consequence of the Lorentz transformation properties of the vector-fields without any explicit momentum dependence of the scalar and vector fields. In fact, only very small momentum dependence has appeared in the relativistic Hartree-Fock (RHF) calculation [9, 10].

In the high energy region, however, the vector-fields must become very small to explain the optical potential of the proton-nucleus elastic scattering [2, 11], and the transverse flow in the heavy-ion collisions [12]. The momentum-dependent part is not actually small though it has not been clearly seen in the low energy phenomena. Furthermore S. Typel [13] introduces the non-local parts and succeeded to improved nuclear properties.

In the previous paper [14] we showed that the momentum dependence of the Dirac-fields is very sensitive to the Fermi velocity though it affects little the nuclear equation of state. In that work we introduced the one-pion exchange force, which produce the dominant contribution of the momentum dependence and suppresses the Fermi velocity, and explain the ISGQR energy.

We can easily imagine that the one-pion exchange force largely produce the momentum dependence because the interaction range is largest. Since the momentum-dependent fields break the current conservation, we have to define the new current caused by the vertex correction.

In this paper, thus, we investigate the nuclear current using the momentum-dependent Dirac fields. For this purpose we define a new current to hold the current conservation in the momentum-dependent Dirac fields, and discuss its effect on the the nuclear static current. In this work we focus only on the convection current which is sensitive to the Fermi velocity, and omit the spin current.

In the next section we explain our formalism to make a conserved current under the momentum-dependent selfenergies. In Sec. 3 we show our numerical results for the static current in our formulation. Then we summarize our work in Sec. 4.

II. FORMALISM

A. Nucleon Propagator

Now we describe the propagator of nucleon with momentum p in the isospin space as follows.

$$S(p) = \begin{pmatrix} S_p(p) & 0 \\ 0 & S_n(p) \end{pmatrix} \tag{1}$$

Here we assume the spin isospin-saturated nuclear matter, and define the proton and neutron propagator as

$$S_N(p) = S_p(p) = S_n(p). \tag{2}$$

The nucleon propagator in the selfenergy Σ is given by

$$S_N^{-1}(p) = \not{p} - M - \Sigma(p), \tag{3}$$

where $\Sigma(p)$ has a Lorentz scalar part U_s and a Lorentz vector part $U_\mu(p)$ as

$$\Sigma(p) = -U_s(p) + \gamma^\mu U_\mu(p). \tag{4}$$

For the future convenience we define the effective mass and the kinetic momentum as

$$\begin{aligned} M^*(p) &= M - U_s(p), \\ \Pi_\mu(p) &= p_\mu - U_\mu(p). \end{aligned} \tag{5}$$

The single particle energy with momentum $\boldsymbol{p}$ is obtained as

$$\begin{aligned} \varepsilon(\boldsymbol{p}) &= p_0|_{on-mass-shell} \\ &= \sqrt{\Pi^2(\boldsymbol{p}) + M^{*2}} + U_0(\boldsymbol{p}). \end{aligned} \tag{6}$$

Then the detailed form of the nucleon propagator eq. (3) is represented by

$$S_N(p) = S_F(p) + S_D(p) \tag{7}$$

with

$$S_F(p) = \{\not{\Pi}(p) + M^*(p)\}\frac{1}{\Pi^2 - M^{*2} + i\delta} \tag{8}$$

$$S_D(p) = 2i\pi\{\not{\Pi}(p) + M^*(p)\}n(\boldsymbol{p})\theta(p_0)\delta[V(p)], \tag{9}$$

where $n(\boldsymbol{p})$ is the momentum distribution, and

$$V(p) \equiv \frac{1}{2}\{\Pi^2(p) - M^{*2}(p)\}. \tag{10}$$

B. Momentum-Dependent Selfenergies

We can easily suppose that it is the one-pion exchange force which produces the major momentum dependence because the interaction range is largest. In this work, thus, we introduce the momentum dependence to the Dirac fields due to the one-pion exchange, and discuss how the Fock parts affects the nuclear current.

Along this line we define a Lagrangian density in the system as

$$\begin{aligned}\mathcal{L} \;=\; & \overline{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a - \frac{1}{2}m_\pi^2 \phi_a \phi_a - \widetilde{U}[\sigma] + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu \\ & \rho_{\mu a}\rho_a^\mu + i\frac{f_\pi}{m_\pi}\overline{\psi}\gamma_5\gamma^\mu \tau_a \psi \partial_\mu \phi_a + g_\sigma \bar{\psi}\psi\sigma - g_\omega \bar{\psi}\gamma_\mu \psi \omega^\mu - \frac{C_v^{IV}}{2M^2}\{\bar{\psi}\gamma_\mu \tau \psi\}^2 \end{aligned} \tag{11}$$

where ψ, ϕ, σ and ω are the nucleon, pion, sigma-meson and omega-meson fields, respectively, and the suffix a indicates the isospin component. In the above expression we use the pseudo-vector coupling form as an interaction between nucleon and pion. The selfenergy potential of the σ-field $\widetilde{U}[\sigma]$ is given as Ref. [12, 15].

$$\widetilde{U}[\sigma] = \frac{\frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{3}B_\sigma \sigma^3 + \frac{1}{4}C_\sigma \sigma^4}{1 + \frac{1}{2}A_\sigma \sigma^2} \; . \tag{12}$$

The symbols m_π, m_σ and m_ω are the masses of π-, σ- and ω-mesons, respectively. In addition, we also introduce the isovector nucleon-nucleon interaction into the Lagrangian (11). so as to discuss on the isovector current later.

Next we calculate the nucleon selfenergies. The nucleon selfenergies are separated into the local part and the momentum-dependent part as $U_\alpha(p) = U_\alpha^L + U_\alpha^F(p)$, where $\alpha = s, \mu$. The σ- and ω-meson exchange parts produce only very small momentum dependence of nucleon selfenergies [9, 10] as their masses are large. In fact the RH and RHF approximations do not give any different results in nuclear matter properties after fitting parameters of σ- and ω-exchanges [9]. On the other hand the one-pion exchange force is a long range one and makes for a large momentum dependence while it does not contribute to the local part in the spin-saturated system. Subsequently we make the local part by RH of the σ- and ω-meson exchanges, and the momentum-dependent part by RHF of the pion exchange, and thus we omit the kinetic energy part mesons except pion in eq.(11). This method is shown in Ref.[11] to keep the self-consistency within the RHF framework.

In this model the local part of the selfenergies are given as

$$U_s^L = g_\sigma \langle\sigma\rangle \tag{13}$$

$$U_\mu^L = \delta_{0\mu} \frac{{g_\omega}^2}{m_\omega^2} \rho_H \tag{14}$$

where $\langle\sigma\rangle$ is the scalar mean-field obtained as

$$\frac{\partial}{\partial\langle\sigma\rangle} \tilde{U}[\langle\sigma\rangle] = g_\sigma \rho_s \tag{15}$$

In the above equations the scalar density ρ_s and the vector Hartree density ρ_H are given by

$$\rho_s = 4 \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \frac{M_\alpha^*(p)}{\tilde{\Pi}_0(p)}, \tag{16}$$

$$\rho_H = 4 \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_0(p)}{\tilde{\Pi}_0(p)}, \tag{17}$$

where $n(\boldsymbol{p})$ is the momentum-distribution, and $\tilde{\Pi}_\mu(p)$ is defined by

$$\tilde{\Pi}_\mu(p) = \frac{1}{2} \frac{\partial}{\partial p^\mu} [\Pi^2(p) - M^{*2}(p)] \tag{18}$$

As a next step we define the momentum-dependent parts of the selfenergies as the Fock parts with the one-pion exchange. When using the pseudo vector (PV) coupling the Fock parts do not become zero at the infinite limit of the momentum $|\boldsymbol{p}|$. One usually erases these contributions by introducing the cut-off parameter. In this work, instead of that, we subtract these contributions from the momentum-dependent parts (these contributions can be renormalized into the Hartree parts): $U_\alpha \to U_\alpha - U_\alpha(p \to \infty)$. Thus we obtain the momentum-dependent parts of the selfenergies as

$$U_s^F(p) = \frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{19}$$

$$U_\mu^F(p) = -\frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{20}$$

where the $\Delta_\pi(q)$ is the pion propagator defined as

$$\Delta_\pi(q) = \frac{1}{q^2 - m_\pi^2}. \tag{21}$$

In the above vector selfenergies we omit the tensor-coupling part involving $[\Pi(k)\cdot(p-k)](p-k)_\mu$. This term is very small if the selfenergy is independent of momentum [9], and their momentum dependence is actually very small as shown later.

C. One-Body Current Operator

If the selfenergy has a momentum dependence, the current operator must be also changed to satisfy the current conservation. We then define the current vertex $\Gamma^{\mu}(p+q,p)$ as

$$\Gamma^{\mu}(p+q,p) = \gamma^{\mu} + \Lambda^{\mu}(p+q,p). \tag{22}$$

The Ward-Takahashi (WT) identity gives the following relation about the current vertex

$$S_N(p+q)q^{\mu}\Gamma_{\mu}S_N(p) = -S_N(p+q) + S_N(p). \tag{23}$$

This expression is rewritten as

$$q^{\mu}\Gamma_{\mu} = S_N^{-1}(p+q) - S_N^{-1}(p). \tag{24}$$

Substituting eq.(3) into the above equation (24), the density-dependent vertex correction Λ^{μ} is obtained as

$$q_{\mu}\Lambda^{\mu}(p+q,p) = -\Sigma(p+q) + \Sigma(p). \tag{25}$$

In this work we restrict our discussion on the convection current, not on the spin current. In Appendix A we show an approximate method to derive the vertex correction from the above WT identity though the anomalous current, which is proportional to $\sigma_{\mu\nu}q^{\nu}$, still has an ambiguity because the WT identity cannot make any condition for it.

When we use the one-pion exchange force, we have to take into account the meson-exchange current. In Appendix B we show that our approximate formulation is also satisfied in the electromagnetic current by including the one-pion exchange current, and the electromagnetic current operator is proportional to $(1+\tau_3)/2$, namely only protons contribute to the convection current. In this work we focus on the static current in the nuclear matter, then we need to get the zero limit of the momentum transfer q. In this limit the vertex correction becomes

$$\Lambda^{\mu}(p) = \lim_{q\to 0}\Lambda^{\mu}(p+q,p) = -\frac{\partial}{\partial p_{\mu}}\Sigma(p). \tag{26}$$

Using the above vertex correction, the current density of the whole system is given as

$$\begin{aligned}
\boldsymbol{j}_{\mu} &= \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\{\frac{1+\tau_3}{2}(\gamma_{\mu} - \frac{\partial\Sigma}{\partial p_{\mu}})S(p)\} \\
&= \int \frac{d^4p}{(2\pi)^3} n^{(p)}(\boldsymbol{p})\{\Pi_{\mu}(p) - \Pi^{\nu}(p)\frac{\partial U_{\nu}(p)}{\partial p_{\mu}} + M^*(p)\frac{\partial U_s(p)}{\partial p_{\mu}}\}\delta[V(p)] \\
&= \int \frac{d^3p}{(2\pi)^3} n^{(p)}(\boldsymbol{p})\left.\frac{\tilde{\Pi}_{\mu}(p)}{\tilde{\Pi}_0(p)}\right|_{p_0=\varepsilon(\boldsymbol{p})},
\end{aligned} \tag{27}$$

where $n^{(p)}$ is the Fermi distribution function for proton, and $\tilde{\Pi}_\mu$ is defined by

$$\begin{aligned}\tilde{\Pi}_\mu(p) &\equiv \frac{\partial}{\partial p^\mu}V(p) \\ &= \frac{1}{2}\frac{\partial}{\partial p^\mu}\{\Pi^2(p) - M^{*2}(p)\}.\end{aligned} \tag{28}$$

Let us consider the one-particle state on the Fermi surface. The space current density contributed from this nucleon can be written as

$$\boldsymbol{j} = \frac{\tilde{\Pi}(p)}{\tilde{\Pi}_0(p)}|_{|\boldsymbol{p}|=p_F} = \boldsymbol{D}_{\boldsymbol{p}}\varepsilon(\boldsymbol{p})|_{|\boldsymbol{p}|=p_F}, \tag{29}$$

where the total derivative $\boldsymbol{D}_{\mathbf{p}}$ is defined on the on-mass-shell condition: $p_0 = \varepsilon(\boldsymbol{p})$. The above equation completely agrees with that derived by the semi-classical way [11].

In the non-relativistic framework the effective mass is defined by

$$M_L^* = (2\frac{d}{d\boldsymbol{p}^2}\varepsilon(\boldsymbol{p}))^{-1}|_{|\boldsymbol{p}|=p_F}, \tag{30}$$

which is so called the '**Landau mass**'. Then the above spatial current density is

$$\boldsymbol{j} = \frac{\boldsymbol{p}_F}{M_L^*}. \tag{31}$$

In our case including the momentum-dependent Dirac fields, the value of the Landau mass M_L^* cannot be uniquely determined from the relativistic effective mass M^* while in the Hartree approximation the Landau mass becomes $M_L^* = \Pi_0(\mathbf{p}_F)$.

D. Core Polarization Current

As for the actual nuclear current observed by experiments the core polarization also plays an important role. Of course this effect cancels contribution of the effective mass in the isoscalar case [16].

Here we should consider a system that one valence nucleon populates a state on the Fermi surface of the saturated nuclear matter. In this system the momentum-distribution can be described as

$$n(\boldsymbol{p}, \tau) = n_0(\boldsymbol{p}) + \frac{1}{4}\Delta n(\boldsymbol{p}, \tau), \tag{32}$$

where $n_0(\boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|)$ shows the usual Fermi distribution with the Fermi momentum p_F, and $\Delta n(\boldsymbol{p}) \propto \delta(|\boldsymbol{p}| - p_F)$ indicates the valence nucleon part. The suffix τ decades the isospin for the valence nucleon,

The valence nucleon varies the selfenergies of nucleons below Fermi surface from that at the saturated matter as

$$U_\alpha(p) \rightarrow U_\alpha(p) + \Delta U_\alpha(p). \tag{33}$$

In addition the function $V(p)$ is also varied as

$$V(p) = V_0(p) + \Delta V(p) \tag{34}$$

with

$$\Delta V(p) = -\Pi^\mu \Delta U_\mu + M^* \Delta U_s. \tag{35}$$

The current density is described with the following expression.

$$j^\mu_{tot} = -\sum_{\tau=\pm 1} \int \frac{d^4p}{(2\pi)^3} f(p,\tau) \frac{\partial V(p)}{\partial p^\mu}, \tag{36}$$

where $f(p,\tau)$ is the four-dimensional momentum-distribution for nucleon with isospin τ, which is given as

$$\begin{aligned} f(p,\tau) &= n(\boldsymbol{p},\tau)\delta(p_0 - \varepsilon_{\boldsymbol{p}}) && (37)\\ &= \frac{1}{\tilde{\Pi}_0(p)} n(\boldsymbol{p},\tau)\delta[V(p)]\theta(p_0). && (38)\end{aligned}$$

The variation along eqs. (32) $-$ (35) leads to the above four dimensional momentum-distribution $f(p,\tau)$ as

$$f(p,\tau) = f_0(p,\tau) + \Delta f(p,\tau) \tag{39}$$

with

$$\begin{aligned} f_0(p,\tau) &= n_0(\boldsymbol{p})\delta[V_0(p)]\theta(p_0), && (40)\\ \Delta f(\boldsymbol{p},\tau) &= \Delta n(\boldsymbol{p},\tau)\delta[V_0(p)]\theta(p_0) + n_0(\boldsymbol{p}) \left\{ \frac{\partial \delta[V(p)]}{\partial V} \right\}_{V=V_0} \Delta V(p,\tau)\theta(p_0). && (41)\end{aligned}$$

The first term comes from the valence nucleon, and the second one from the core polarization. The total current density is given as

$$\begin{aligned} j^\mu_{tot} &= \int \frac{d^4p}{(2\pi)^3} f(p)\delta[V(p)] \frac{\partial V(p)}{\partial p_\mu} && (42)\\ &= \delta^\mu_0 \rho_B + j^\mu_{val} + j^\mu_{cor}. && (43)\end{aligned}$$

The first term is the current density of the saturated matter, and the second current density j^{μ}_{val} shows the contribution from the valence nucleon as

$$j^{\mu}_{val} = \int \frac{d^4p}{(2\pi)^3} \Delta n(\boldsymbol{p}) \delta(p_0 - \varepsilon_{\boldsymbol{p}}) \frac{\tilde{\Pi}_{\mu}(p)}{\tilde{\Pi}_0(p)}. \tag{44}$$

The third current density j^{μ}_{cor} is so called the core polarization current density, which is caused by the variation of the selfenergies of nucleons in Fermi sea and given by

$$\begin{aligned} j^{\mu}_{cor} &= -2 \sum_{\tau=\pm 1} \int \frac{d^4p}{(2\pi)^3} n_0(\boldsymbol{p},\tau) [\frac{\partial \Delta V(p)}{\partial p_{\mu}} \delta[V_0(p)] + \frac{\partial V_0(p)}{\partial p_{\mu}} \{ \frac{\partial \delta[V(p)]}{\partial V} \}_{V=V_0} \Delta V(p)] \quad (45) \\ &= -2 \sum_{\tau=\pm 1} \int \frac{d^4p}{(2\pi)^3} n_0(\boldsymbol{p},\tau) \frac{\partial}{\partial p^{\mu}} \{ \Delta V(p) \delta[V_0(p)] \}. \quad (46) \end{aligned}$$

Here it should be noted that the time component of the core polarization current density does not change the nucleon density:

$$j^{0}_{cor} = -2 \sum_{\tau=\pm 1} \int \frac{d^4p}{(2\pi)^3} n_0(\boldsymbol{p},\tau) \frac{\partial}{\partial p^0} \{ \Delta V(p) \delta[V_0(p)] \} = 0. \tag{47}$$

Now we define the z-axis as the direction of the current at the matter. First we calculate the isoscalar current density by taking the valence nucleon part of the momentum distribution to be

$$\Delta n(\boldsymbol{p},\tau) = \frac{(2\pi)^3}{\Omega} \delta(\boldsymbol{p} - \boldsymbol{a}) \tag{48}$$

with

$$\boldsymbol{a} = p_F \hat{z}, \tag{49}$$

where Ω is the volume of the system, which should be finally taken to be infinite. The core polarization current density becomes

$$\begin{aligned} j^{3}_{cor} &= -4 \int \frac{d^4p}{(2\pi)^3} \theta(p_F - |\boldsymbol{p}|) \frac{\partial}{\partial p_z} \{ \Delta V(p) \delta[V_0(p)] \} \quad (50) \\ &= -4 \int \frac{d^4p}{(2\pi)^3} \delta(p_F - |\boldsymbol{p}|) \frac{p_z}{p_F} \{ \Delta V(p) \delta[V_0(p)] \} \quad (51) \\ &= -\frac{1}{2\pi^3} \int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{\Delta V(p)}{\tilde{\Pi}_0(p)}. \quad (52) \end{aligned}$$

Then we separate it to several parts as

$$j^{3}_{cor} = j^{3}_{cor}(H) + j^{3}_{cor}(F) \tag{53}$$

with

$$j^3_{cor}(H) \;=\; -\frac{1}{2\pi^3}\int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{-\Pi^\mu \Delta U^H_\mu + M^* \Delta U^H_s}{\tilde{\Pi}_0(p)} \tag{54}$$

$$j^3_{cor}(F) \;=\; -\frac{1}{2\pi^3}\int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{-\Pi^\mu \Delta U^F_\mu + M^* \Delta U^F_s}{\tilde{\Pi}_0(p)}, \tag{55}$$

where $\Delta U^H_{\mu(s)}$ and $\Delta U^F_{\mu(s)}$ are shown to be contributions of $\Delta U_{\mu(s)}$.from Hartree and Fock parts of selfenergies, respectively.

It is not so easy to solve the above equation exactly in the RHF case though it is possible in the RH case. On the other hand we have known that the actual momentum dependence is very small at least below the Fermi momentum. Then we can suppose that a perturbative way is possible with the respect to the momentum dependence.

Before explaining the actual method, first, we would like to explain the relativistic Hartree (RH) case. There the selfenergies are momentum-independent, and the valence current density becomes

$$j^3_{var} = \frac{1}{\Omega}\frac{p_F}{E^*_F}. \tag{56}$$

In this case the core-polarization current density is calculated in the following way.

$$\begin{aligned} j^3_{cor} \;&=\; -\frac{1}{2\pi^3}\int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{\Pi_z \Delta U^H_z}{E^*_F} \\ &=\; -\frac{4}{3\pi^2}\frac{p_F^3}{E^*_F}\Delta U^H_z \end{aligned} \tag{57}$$

In the RH calculation

$$\begin{aligned} \Delta U^H_z \;&=\; \frac{g_v^2}{m_v^2}\int \frac{d^3p}{(2\pi)^3} n(\boldsymbol{p}) \frac{p_z}{E^*_p} \\ &=\; \frac{g_v^2}{m_v^2} j^3. \end{aligned} \tag{58}$$

Substituting eq.(58) into eq.(57), we can get

$$\begin{aligned} j^3 \;&=\; \frac{1}{\Omega}\frac{p_F}{E^*_F} - \{\frac{g_v^2}{m_v^2}\frac{4}{3\pi^2}\frac{p_F^3}{E^*_F}\} j^3 \\ &=\; \frac{1}{\Omega}\frac{p_F}{E^*_F}\{1 + \frac{g_v^2}{m_v^2}\rho_B \frac{1}{E^*_F}\}^{-1} \end{aligned} \tag{59}$$

In the RH case the Fermi energy is obtained as

$$\varepsilon_F = E^*_F + \frac{g_v^2}{m_v^2}\rho_B, \tag{60}$$

and then

$$j^3 = \frac{1}{\Omega}\frac{p_F}{\varepsilon_F}. \tag{61}$$

In the low density region below about the saturation, $\varepsilon_F \approx M$, so that we can see that the core polarization plays a role to cancel the effect of the effective mass in the valence current density. In the RHF case the contribution from the Hartree part is large, and we cannot use the perturbative way. Since momentum dependence of the selfenergies is not so large, however, the difference between $\tilde{\Pi}_0$ and Π_0 is small, and then the Hartree part of the total current density $j^3(H)$ can be approximately gotten with the following equation.

$$\begin{aligned} j^3(H) &\approx \int \frac{d^3p}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_z(p)}{\tilde{\Pi}_0(p)} \\ &\approx \int \frac{d^3p}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_z(p)}{\Pi_0(p)} \\ &\approx j^3_{var}(H) - \Delta U_z \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \{ \frac{\partial}{\partial U_z} \frac{\Pi_z(p)}{\Pi_0(p)} \}_{\Delta U_z = 0} \\ &\approx j^3_{var}(H) - \Delta U_z \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \frac{\Pi_z(p)}{\Pi_0(p)} \{ 1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)} \}, \end{aligned} \tag{62}$$

where $j^3_{var}(H)$ is the valence part of the Hartree current density as

$$j^3_{var}(H) \approx \frac{1}{\Omega}\frac{\Pi_z(p_F)}{\tilde{\Pi}_0(p_F)}. \tag{63}$$

The space component of the vector selfenergy, which is caused only by the Fock contribution in the saturation matter, is very small, and then ΔU_z is thought to be contributed from the Hartree parts as

$$\Delta U_z \approx \Delta U_z^H = \frac{g_v^2}{m_v^2} j^3(H). \tag{64}$$

Then the Hartree contribution of the core polarization current density is approximately given as

$$j^3_{cor}(H) = j^3(H) - j^3_{var}(H) \approx \frac{-V_C^H(IS)}{1 + V_C^H(IS)} j^3_{var}(H) \tag{65}$$

with

$$V_C^H(IS) = \frac{g_v^2}{m_v^2} \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \frac{1}{\tilde{\Pi}_0} \{ 1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)} \}. \tag{66}$$

As for the Fock part, the momentum dependence of selfenergies are not so large, and its contribution is not so big in the total current density. Instead of getting it exactly, thus, we can use

the perturbative way for the Fock part of the core polarization current density. Along this line the variation of the selfenergies are taken to be only the contribution from the valence nucleon as

$$\Delta U_s^F(p) \approx \frac{3f_\pi^2}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Delta n(\boldsymbol{k})\frac{M^*(k)}{\tilde{\Pi}_0(k)}\Delta_\pi(p-k), \tag{67}$$

$$\Delta U_\mu^F(p) \approx -\frac{3f_\pi^2}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Delta n(\boldsymbol{k})\frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)}\Delta_\pi(p-k). \tag{68}$$

Then we substitute them into the eq. (55), and get

$$j^3_{cor}(F) = \frac{3f_\pi^2}{4\pi^3}\tau_3 \int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{\Pi_0^2(p_F) - \Pi_v^2(p_F) + M^{*2}(p_F)}{\tilde{\Pi}_0^2(p_F)}\Delta_\pi(0;\boldsymbol{p}-\boldsymbol{a}). \tag{69}$$

Next we consider the isovector current. In the similar way we can calculate the isovector current density by taking the variation part of the momentum distribution as

$$\Delta n(\boldsymbol{p},\tau) = \frac{(2\pi)^3}{\Omega}\delta(\boldsymbol{p}-\boldsymbol{a}). \tag{70}$$

In this work the nuclear system is taken to be the isospin symmetric saturated matter plus valence nucleon. Thus, the isovector properties can be treated in the perturbative way. Namely it can be considered that the Dirac-fields of the valence nucleon isoscalar one, and that those of the nucleon in Fermi sea has a very small isovector part coming from the valence nucleon.

As for the Hartree part we substitute the following $V_C^H(IV)$ instead of $V_C^H(IS)$ into the equation (65):

$$V_C^H(IV) = \frac{C_v^{IV}}{M^2}\int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p})\frac{1}{\tilde{\Pi}_0}\{1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)}\}. \tag{71}$$

As for the Fock part, furthermore, the variations of the selfenergies become

$$\Delta U_s^F(p) \approx \tau_3\frac{f_\pi^2}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Delta n(\boldsymbol{k})\frac{M^*(k)}{\tilde{\Pi}_0(k)}\Delta_\pi(p-k), \tag{72}$$

$$\Delta U_\mu^F(p) \approx -\tau_3\frac{f_\pi^2}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\Delta n(\boldsymbol{k})\frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)}\Delta_\pi(p-k). \tag{73}$$

Then the Fock contribution of the isovector core polarization current density is obtained as

$$j^3_{cor}(F) = \frac{f_\pi^2}{4\pi^3}\tau_3 \int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \frac{\Pi_0^2(p_F) - \Pi_v^2(p_F) + M^{*2}(p_F)}{\tilde{\Pi}_0^2(p_F)}\Delta_\pi(0;\boldsymbol{p}-\boldsymbol{a}). \tag{74}$$

III. RESULTS

In this section we show results calculated with the above formulation. In this calculation we use the parameters (PF1) [14] for the σ- and ω- exchanges to reproduce the saturation properties of nuclear matter: the binding energy $BE = 16$MeV, the incompressibility $K = 200$MeV and the effective mass $M^*/M = 0.7$ at the saturation density $\rho_0 = 0.17\mathrm{fm}^{-3}$. For comparison we give results with momentum-independent selfenergies obtained by the parameter-set PM1 [15] that gives the same saturation properties. As for the isovector nucleon-nucleon interaction, C_v^{IV}, we take the value of PM1. These values are written in Table 1.

In Fig. 1 we draw the momentum dependence of the scalar selfenergy $U_s(p)$ and that of the time component of the vector selfenergy $U_0(p)$. It can be seen that the variation of the momentum-dependent selfenergies is only 2.5 % at most below Fermi level, which looks very small.

In Fig. 2 we show the density-dependence of the Dirac selfenergies U_s and U_0 on the Fermi-surface (a) and the Landau mass (b) with the parameter-sets, PF1 and PM1. Though two results of U_s and U_0 almost agree each other, we can see rather large difference in the Landau mass: the value at $\rho_B = \rho_0$ is $M_L^*/M = 0.85$ in PF1 which is consistent with the value expected by the analysis of ISGQR as shown previously. On the contrary, the momentum-independent calculation (PM1) gives $M_L^*/M = 0.74$ which overestimates the excitation energy of ISGQR. Hence it is shown that the very small momentum dependence in the nucleon selfenergies enhances the Fermi velocity about 15 %, and gives a significant difference in the Landau mass.

Furthermore we can also see an interesting behaviour of M_L^* in PF1, namely, its value agrees with the bare mass at $\rho_B \approx 0.5\rho_0$ and becomes larger with the decrease of the density. Effects of small Dirac effective mass are largely cancelled at low density by the momentum dependence created by the one-pion exchange.

In Fig. 3 we show the density dependence of the isoscalar current density. In the upper column (Fig. 3a) the solid and chain-dotted lines indicate the total current density and the valence current density, respectively. For comparison the current density for the RH approximation are also drawn there with the dashed line. From that we can know that the Fock contribution suppresses the RH current density, and the core polarization further suppresses it.

In the lower column (Fig. 3b) we show the contribution from the core polarization. The long dashed and dotted lines indicate the core polarization current density contributed from the Hartree and Fock parts, respectively. The Hartree contribution reduces the current density,

while the Fock contribution enhances it.

In Fig. 4 we show the isovector current densities; the meaning of each line is the same as that in Fig. 3. As for the isovector channel the core polarization does not affect the total current density noticeably.

The most direct observable for the nuclear static current must be the magnetic moment; the nuclear medium effect is examined as the discrepancy from the Schmidt value. Thus we should compare our results with the normal current density, which is a current density with no medium effect and given as

$$j_o^3 = v_F = \frac{p_F}{\varepsilon_F}. \tag{75}$$

Here we define the following quantity as

$$\Delta j_r^3 = \frac{j_{tot}^3 - j_o^3}{j_o^3}. \tag{76}$$

In Fig. 5 we show the density dependence of Δj_r^3. As for the isoscalar current the total curren t density, j_{tot}^3, almost agrees with the normal current density, j_o^3. This result is consistent with the fact that the core polarization cancels the effect caused by the effective mass. As for the isovector, on the other hand, the core polarization does not have significant effect. Here we can see very interesting result that the total current density is 10 % less than the normal current density in low density region around $\rho_B \approx \rho_0/4$. This result is consistent with the experimental fact that the isovector magnetic moment is 10 % less than the Schmidt value; here we should note that the magnetic moment indicates the medium effect in surface region. Of course our calculation is performed for the infinite matter, and does not include the contribution from the anomalous part. The result does not directly show the experimental observable, but this results is very suggestible.

IV. CONCLUDING REMARKS

In this paper we have defined a current operator which is consistent with the momentum-dependent Dirac fields. It was shown that this current operator automatically include the exchange current. This current operator describes the static spatial current which is determined by the Landau mass M_L^* independently of the effective mass M^*. This fact is satisfied in all cases though we here show it only in the one-pion exchange case which is the most effective.

Furthermore we calculate the static current in the system with one valence nucleon on the Fermi surface of the saturated nuclear matter. In this calculation we introduce the core polarization effect. We can confirm that the core polarization cancels the enhancement caused by the effective mass in the isoscalar current, but hardly affects the isovector current.

As shown in the Ref. [14], the very small momentum dependence in the nucleon selfenergies enhances the Fermi velocity, even if this momentum dependence is negligibly small for the nuclear EOS; in the present calculation the Fermi velocity is enhanced 15 % by the momentum dependence caused by the one-pion exchange. Then we succeed to reduce the isovector current density in low density region; the value of the current density is almost equivalent to the current density without effective mass at $\rho_B \approx 0.5\rho_0$, and 10% suppressed around $\rho_B \approx 0.25\rho_0$. The latter result is consistent with that the observed isovector magnetic moment is 10% smaller than the Schmidt value; of course the quantitative conclusion has not been so clear.

As seen in this paper the momentum-dependent parts, which are non-local in the finite nuclei, are very effective in observables related with Fermi velocity even if these parts are small. In future we need to discuss effects of the non-local parts of Dirac fields to study nuclear structure and reactions.

The typical value of effective mass is empirically known as $M_N^*/M_N = 0.55 - 0.7$ [2, 3, 4, 19, 20]. If we use other parameter-set which give smaller effective mass than ours, the effects of the momentum-dependent part created by the one-pion exchange does not have a sufficient effect to explain the fermi velocity expected from experimental analysis. Exchange forces of other mesons, σ, ω, η and δ, also contribute to suppressing the Fermi velocity if we choose the PV-coupling for π- and η- nucleon coupling.

Here we should give a further comment. Bentz et al. have shown in Ref. [17] that the Landau mass is reduced by the one-pion exchange, which is opposite to ours. This result is consistent with the nonrelativistic analysis on the magnetic moment with the exchange current [21]. In this work Miyazawa showed that the exchange current enhances the convection part, reduces the spin part, and totally reduces the isovector magnetic moment. In the calculation [17] Bentz et al. have used the pseudo-scalar (PS) coupling, and the sign of U_μ^{MD} was taken to be opposite to ours. The full HF calculation with the PS coupling makes too large contribution to the Dirac selfenergies [3] while Bentz et al. calculated the Fock term with a perturbative way. Thus a calculation with the PV coupling must be more reliable than that with the PS coupling.

The large discrepancy between the PS and PV coupling comes from relativistic effects in the

one-pion exchange. Since the pion mass is smaller than the nucleon fermi energy, relativistic effects must be larger in the one-pion exchange. Miyazawa treated the one-pion exchange in the nonrelativistic way. Thus it is not strange that our results qualitatively disagree with Miyazawa's one.

In this work we calculate and discuss only the convection current, but not spin current. However our formalism is satisfied for the whole Dirac current, thus the spin-current except for the anomalous current must be affects by the momentum dependence of the selfenergy in the same way. As for the anomalous current we have to consider the vertex correction in another way. Since there is no connection between the upper and lower components of the Dirac spinor in this current, we can suppose that a perturbative treatment gives sufficient results. It is one of the future works.

Since the Fock effects are largely seen in the wide energy region [11, 12], the new current we suggested here also plays an important role with the large momentum-transfer q. In future we need to discuss effects of this current in the high momentum transfer phenomena such as the quasielastic electron scattering [22].

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APPENDIX A: ONE-BODY CURRENT OPERATOR

The density-dependent vertex correction Λ^μ is obtained as

$$q_\mu \Lambda^\mu(p+q,p) = -\Sigma(p+q) + \Sigma(p). \tag{A1}$$

Within the one-boson exchange force, the Fock part of the selfenergy is generally written in the following way.

$$\begin{aligned}\Sigma_F(p) &= i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S_N(k)\gamma_a \Delta^{(a)}(p-k) \\ &+ i\sum_b \tilde{C}_a \int \frac{d^4k}{(2\pi)^4}[(\not{p}-\not{k}),\gamma^b]S_N(k)[\gamma_b,(\not{p}-\not{k})]\Delta^{(b)}(p-k)\end{aligned} \tag{A2}$$

where $\gamma_{a(b)}$ is the γ-matrix with the suffix $a(b)$ indicating the scalar, pseudo-scalar, vector, axial-vector and tensor, and $\Delta^{(a)}$ is the propagator of meson with the quantum number indicated with the suffix a.

Substituting eq. (A2) into eq. (A1), we get

$$\begin{aligned}q_\mu \Lambda^\mu(p+q,p) &= -\Sigma(p+q) + \Sigma(p) \\ &= i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S_N(k)\gamma_a\{\Delta^{(a)}(p-k+q) - \Delta^{(a)}(p-k)\} \\ &+ i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}\{[(\not{p}-\not{k}+\not{q}),\gamma^b]S_N(k)[\gamma_b,(\not{p}-\not{k}+\not{q})]\Delta^{(b)}(p-k+q) \\ &\qquad - [(\not{p}-\not{k}),\gamma^b]S_N(k)[\gamma_b,(\not{p}-\not{k})]\Delta^{(b)}\}\end{aligned} \tag{A3}$$

When we omit the vertex form factor of the meson-nucleon coupling, the meson propagator is given as

$$\Delta_a(k) = \frac{1}{k^2 - m_a^2}, \tag{A4}$$

and

$$\Delta^{(a)}(k+q) - \Delta^{(a)}(k) = -\Delta^{(a)}(k+q)q(2k+q)\Delta^{(a)}(k). \tag{A5}$$

Then the above equation can be rewritten as the following expression,

$$\begin{aligned} q^{\mu}\Lambda_{\mu} &= q^{\mu} i \sum_a C_a \int \frac{d^4k}{(2\pi)^4} \gamma^a S_N(k)\gamma_a \{\Delta^{(a)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(a)}(p-k)\} \\ &+ q^{\mu} i \sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4} \{[\gamma^b, (\not{p}-\not{k}+\not{q})] S_N(k) [\gamma_b, (\not{p}-\not{k})] \\ &\qquad \times \Delta^{(b)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(b)}(p-k) \\ &\qquad - [(\not{p}-\not{k}+\not{q}), \gamma^b] S_N(k) [\gamma_b, \gamma_{\mu}] \Delta^{(b)}(p-k+q) \\ &\qquad - [\gamma_{\mu}, \gamma^b] S_N(k) [\gamma_b, (\not{p}-\not{k})] \Delta^{(b)}(p-k)\} \end{aligned} \tag{A6}$$

From the above equation we can get the following vertex correction, which we call $\Lambda^{(1)}$.

$$\begin{aligned} \Lambda^{(1)}_{\mu} &= i \sum_a C_a \int \frac{d^4k}{(2\pi)^4} \gamma^a S_N(k)\gamma_a \{\Delta^{(a)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(a)}(p-k)\} \\ &+ i \sum_b C_b \int \frac{d^4k}{(2\pi)^4} \{[(\not{p}-\not{k}+\not{q}), \gamma^b] S_N(k) [\gamma_b, (\not{p}-\not{k})] \\ &\qquad \times \Delta^{(b)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(b)}(p-k) \\ &\qquad - [(\not{p}-\not{k}+\not{q}), \gamma^b] S_N(k) [\gamma_b, \gamma_{\mu}] \Delta^{(b)}(p-k+q) \\ &\qquad - [\gamma^b, \gamma_{\mu}] S_N(k) [\gamma_b, (\not{p}-\not{k})] \Delta^{(b)}(p-k)\} \end{aligned} \tag{A7}$$

On the other hand the equation (A2) can be rewritten as

$$\begin{aligned} \Sigma_F(p) &= i \sum_a C_a \int \frac{d^4k}{(2\pi)^4} \gamma^a S_N(p-k)\gamma_a \Delta^{(a)}(k) \\ &+ i \sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4} [\not{k}, \gamma^b] S_N(p-k) [\gamma_b, \not{k}] \Delta^{(b)}(k) \end{aligned} \tag{A8}$$

The eq. (A8) is given only by the variable transformation ($k \to p-k$) from (A2). Substituting eq. (A8) into eq. (A1), then, we get

$$\begin{aligned} q_{\mu}\Lambda^{\mu}(p+q,p) &= -\Sigma(p+q) + \Sigma(p) \\ &= i \sum_a C_a \int \frac{d^4k}{(2\pi)^4} \gamma^a \{S_N(p-k+q) - S_N(p-k)\} \gamma_a \Delta^{(b)}(k) \\ &+ i \sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4} [\not{k}, \gamma^b] \{S_N(p-k+q) - S_N(p-k)\} [\gamma_b, \not{k}] \Delta^{(b)}(k) \end{aligned} \tag{A9}$$

Using the WT identity (23), the above equation becomes the following expression.

$$\begin{aligned} q^{\mu}\Lambda_{\mu} &= i q^{\mu} \sum_a C_a \int \frac{d^4k}{(2\pi)^4} \gamma^a S_N(k+q)\Gamma_{\mu} S_N(k) \gamma_a \Delta^{(a)}(p-k) \\ &+ i q^{\mu} \sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4} \{\gamma^b (\not{p}-\not{k}) S_N(k+q)\Gamma_{\mu} S(k) (\not{p}-\not{k}) \gamma_b \Delta^{(b)}(p-k) \end{aligned} \tag{A10}$$

From the above equation we can get another expression of the vertex correction, which we call $\Lambda^{(2)}$, as

$$\begin{aligned}\Lambda^{(2)}_\mu &= i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S_N(k+q)\Gamma_\mu S_N(k)\gamma_a \Delta^{(a)}(p-k) \\ &+ i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}\{[(\not p-\not k),\gamma^b]S_N(k+q)\Gamma_\mu S(k)[\gamma_b,(\not p-\not k)]\Delta^{(b)}(p-k)\end{aligned} \quad (A11)$$

The Feynman diagrams corresponding to the approximate identity $\Lambda^{(2)}_\mu \approx \Lambda^{(1)}_\mu$ are shown in Fig. 6. The diagrams in the upper column and in the lower column indicate the vertex corrections represented with $\Lambda^{(2)}$ and with $\Lambda^{(1)}$, respectively. This approximation rule exhibits that the iteration of the vertex correction is partially equivalent to the meson coupled with the external vector field like the meson exchange current.

The expression of $\Lambda^{(2)}_\mu$ describes the usual vertex correction which is consistent with the Fock selfenergy. However its calculation is not easy to be solved because the equation (A11) includes the iteration scheme. On the other hand we can perform the actual calculation of $\Lambda^{(1)}_\mu$ (A7). In addition we should note that the above expression is satisfied in proton and neutron independently. Thus we should make the following approximation rule:

$$\begin{aligned}& i\int \frac{d^4k}{(2\pi)^4}\gamma^a S_N(k+q)\Gamma_\mu S_N(k)\gamma_a \Delta^{(a)} \approx \\ &\quad i\int \frac{d^4k}{(2\pi)^4}\gamma^a S_N(k)\gamma_a\{\Delta^{(a)}(p-k+q)(2p-2k+q)_\mu \Delta^{(a)}(p-k)\}\end{aligned} \quad (A12)$$

and

$$\begin{aligned}& i\int \frac{d^4k}{(2\pi)^4}\{[(\not p-\not k),\gamma^b]S(k+q)\Gamma_\mu S_i(k)[\gamma^b,(\not p-\not k)] \approx \\ & i\int \frac{d^4k}{(2\pi)^4}\{[(\not p-\not k+\not q),\gamma^b]S_N(k)[\gamma_b,(\not p-\not k)]\Delta^{(b)}(p-k+q)(2p-2k+q)_\mu \Delta^{(b)}(p-k) \\ &\qquad - [(\not p-\not k+\not q),\gamma^b]S_N(k)[\gamma_b,\gamma_\mu]\Delta^{(b)}(p-k+q) \\ &\qquad\qquad - [\gamma^b,\gamma_\mu]S_N(k)[\gamma_b,(\not p-\not k)]\Delta^{(b)}(p-k)\}.\end{aligned} \quad (A13)$$

Here we give a comment. If we substitute the full propagator S_N into eq.(A7). we have to solve the vacuum polarization, which is also very difficult. In the usual RMF approach we usually calculate observables contributed from the nucleon in the Fermi sea by using only the density-dependent part S_D instead of S_F. In the case of the RH, where the selfenergies are

momentum-independent, the following equation is satisfied,

$$
\begin{aligned}
& i\int\frac{d^4k}{(2\pi)^4}\{S_F(k+q)\not{q}S_D(k)\Delta(p-k)+S_D(k)\not{q}S_F(k-q)\Delta(p-k+q)\} \\
= \; & i\int\frac{d^4k}{(2\pi)^4}S_D(k)\{\Delta(p-k+q)-\Delta(p-k)\}.
\end{aligned}
\tag{A14}
$$

This equation mentions us that we can describe particle-hole excitations with the usual approximation that the density dependent part of the nucleon propagator S_D (9) is used in $\Lambda^{(1)}$ instead of the full propagator. The actual momentum dependence is very small, and the equation (A14) is approximately satisfied in the RHF case, too. From that the rules of eqs.(A12) and (A13) can be considered to be available under this approximation.

APPENDIX B: ONE-PION EXCHANGE CURRENT OPERATOR

In this appendix we discuss details of the current operator. As for isoscalar meson-exchanges, the correction term of the current can be directly used in the above $\Lambda_\mu^{(2)}$ or approximately $\Lambda_\mu^{(2)}$. As for isovector meson exchanges, however, the current operator includes the diagrams of the photon connect with the exchange meson (the mesonic current) and that of the photon contact with the meson-nucleon vertex (the contact current).

In the paper we consider only the one-pion exchange, which is considered to be most effective because its mass is smallest in meson masses, and the non-locality of the Fock selfenergy with the pion exchange is largest.

The electro-magnetic interaction Lagrangian density is describe as

$$\mathcal{L}_{em}(x) = \mathcal{L}_{em}^v(x) + \mathcal{L}_{em}^m(x) + \mathcal{L}_{em}^c(x) \tag{B1}$$

with

$$\begin{aligned} \mathcal{L}_{em}^v &= -e\bar{\psi}(x)\gamma_\mu \frac{1+\tau_3}{2}\psi, & \text{(B2)} \\ \mathcal{L}_{em}^m &= -ie\{\phi_1(x)\partial_\mu\phi_2(x) - \phi_2(x)\partial_\mu\phi_1(x)\}, & \text{(B3)} \\ \mathcal{L}_{em}^c &= -\frac{ief_\pi}{m_\pi^2}\{\tilde{\psi}(x)\gamma_\mu\gamma_5\tau_1\psi(x)\phi_2(x) - \tilde{\psi}(x)\gamma_\mu\gamma_5\tau_2\psi(x)\phi_1(x)\ \}. & \text{(B4)} \end{aligned}$$

Now we separate the vertex corrections to three parts $\Lambda_\mu = \Lambda_\mu^v + \Lambda_\mu^m + \Lambda_\mu^c$, which is related with the above three parts of the electromagnetic interactions.

Here we assume that the modified Dirac current is contributed only from the proton as

$$\Gamma_\mu = \gamma_\mu \frac{1+\tau_3}{2} + \Lambda_\mu = \tilde{\Gamma}_\mu \frac{1+\tau_3}{2} \tag{B5}$$

where $\tilde{\Gamma}_\mu$ is an isoscalar operator. The usual vertex correction is given as

$$\Lambda_\mu^v = -\frac{if_\pi^2}{m_\pi^2}\int \frac{d^4k}{(2\pi)^4}(\not{p} - \not{k})\gamma_5\tau_a S(k+q)\tilde{\Gamma}_\mu \frac{1+\tau_3}{2} S(k)\tau_a\gamma_5(\not{p} - \not{k})\Delta(p-k) \tag{B6}$$

Then this equation can be rewritten as

$$\Lambda_\mu^v = \mathcal{J}_\mu(\frac{3}{2} - \frac{1}{2}\tau_3) \tag{B7}$$

with

$$\mathcal{J}_\mu = \frac{if_\pi^2}{m_\pi^2}\int \frac{d^4k}{(2\pi)^4}(\not{p} - \not{k})\gamma_5 S_N(p-k)\tilde{\Gamma}_\mu S_N(k)\gamma_5(\not{p} - \not{k})\Delta_\pi(p-k). \tag{B8}$$

Using the rule given by eq.(A13) the above equation is approximately written as

$$\begin{aligned}
\mathcal{J}_\mu &\approx \tilde{\mathcal{J}}_\mu \\
&= \frac{if_\pi^2}{m_\pi^2}\int \frac{d^4k}{(2\pi)^4}\{\ (\not p - \not k + \not q)\gamma_5 S_N(k)\gamma_5(\not p - \not k)\Delta_\pi(p-k+q)(2p-2k+q)_\mu\Delta_\pi(p-k) \\
&\qquad - (\not p - \not k + \not q)\gamma_5 S_N(k)\gamma_5\gamma_\mu\Delta_\pi(p-k+q) \\
&\qquad - \gamma_5\gamma_\mu S_N(k)\gamma_5(\not p - \not k)\Delta_\pi(p-k)\ \}
\end{aligned} \tag{B9}$$

Next we calculate the contribution Λ^m from the so-called pionic current, where the photon connects with the pion exchange between nucleons,

$$\begin{aligned}
\Lambda_\mu^m &= -\int \frac{d^4k}{(2\pi)^4)}(\frac{if_\pi}{m_\pi})(\not p - \not k + \not q)\gamma_5\tau_i\ (\not p - \not k + \not q)\gamma_5 S_N(k)\gamma_5(\not p - \not k)(\frac{if_\pi}{m_\pi})\tau_j\gamma_5(\not p - \not k) \\
&\qquad \times (\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j})i\Delta(p-k-q)(-i)(2p-2k-q)_\mu i\Delta(p-k) \\
&= -\frac{2if_\pi^2}{m_\pi^2}\tau_3\int \frac{d^4k}{(2\pi)^4)}\{\ (\not p - \not k + \not q)\gamma_5 S_N(k)\gamma_5(\not p - \not k) \\
&\qquad \times \Delta_\pi(p-k+q)(2p-2k+q)_\mu\Delta_\pi(p-k)
\end{aligned} \tag{B10}$$

and the contribution Λ^c from the co-called contact currents or the Siegel current where the photon, pion and nucleon connect at one vertex. These contributions are obtained as

$$\begin{aligned}
\Lambda_\mu^c &= -\int \frac{d^4k}{(2\pi)^4)}\{\ (\frac{if_\pi}{m_\pi})(\not p - \not k + \not q)\gamma_5\tau_i S_N(k)(\frac{if_\pi}{m_\pi})\tau_j\gamma_5\gamma_\mu\Delta_\pi(p-k+q) \\
&\qquad +(\frac{if_\pi}{m_\pi})\gamma_\mu\gamma_5\tau_j S_N(p)\tau_i(\frac{if_\pi}{m_\pi})(\not p - \not k)i\Delta_\pi(p-k+q)\}(\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j}) \\
&= \frac{-2if_\pi^2}{m_\pi^2}\tau_3\int \frac{d^4k}{(2\pi)^4)}\{\ (\not p - \not k + \not q)\gamma_5 S_N(k)\gamma_5\gamma_\mu\Delta_\pi(p-k+q) \\
&\qquad +\ \gamma_5\gamma_\mu S_N(k)\gamma_5(\not p - \not k)\Delta_\pi(p-k)\ \}
\end{aligned} \tag{B11}$$

From eqs.(B10) and (B11) we can obtain

$$\Lambda_\mu^m + \Lambda_\mu^c = 2\tau_3\tilde{\mathcal{J}}_\mu. \tag{B12}$$

Finally the the vertex correction is given as the summation of the above three contributions, which becomes

$$\Lambda_\mu = \Lambda_\mu^v + \Lambda_\mu^m + \Lambda_\mu^c = \frac{3}{2}(1+\tau_3)\tilde{\mathcal{J}}_\mu. \tag{B13}$$

This equation imply that the ansatz shown in eq.(B5) is consistent in the present calculation. Here we would like to give some comments about the above equation. Under the isospin symmetric matter, first, the above vertex correction satisfies the WT identity as

$$\frac{3}{2}q^\mu\tilde{\mathcal{J}}_\mu = \Sigma_F^\pi(p+q) - \Sigma_F^\pi(p) \tag{B14}$$

with

$$\Sigma^{\pi}_{F}(p) = -U^{F}_{s}(p) + \gamma^{\mu} U^{F}_{\mu}(p), \tag{B15}$$

where U^F_s and U^F_μ are the Fock contributions of scalar (19), and vector (20) selfenergies by the one-pion-exchange (19,20), respectively.

	g_σ	g_ω	B_σ	A_σ	f_π	C_v^{IV}
PF1	9.699	9.880	27.61	6.134	1.008	20.32
PM1	9.408	9.993	23.52	5.651	0.0	20.32

TABLE I: Parameter sets in this paper. In all cases have used $m_\pi = 138$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV and $C_\sigma = 0$.

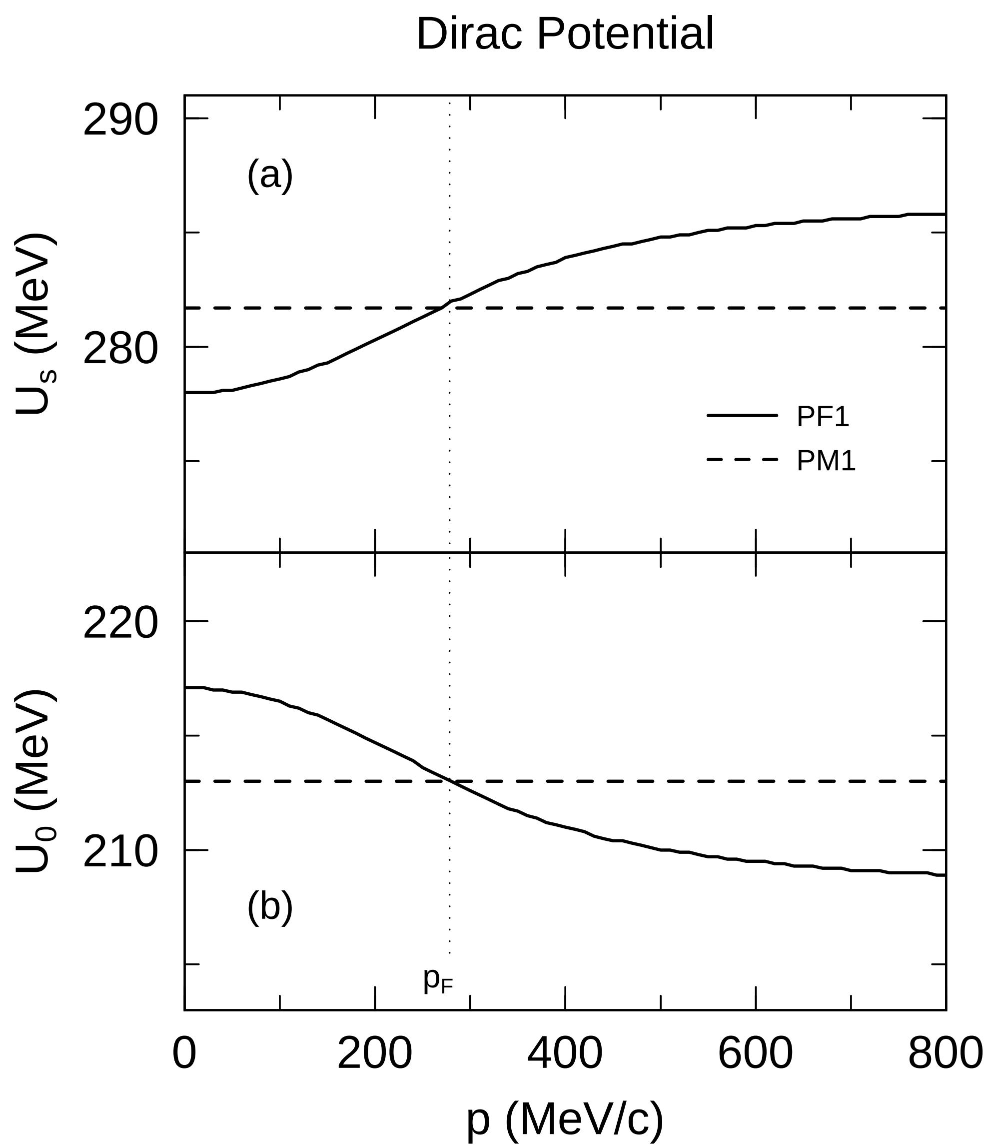

FIG. 1: Momentum-dependence of the scalar (a) and vector (b) selfenergies. The solid and dashed lines indicate the results with PF1 and PM1, respectively. The dotted line denotes the position of the Fermi momentum at $\rho_B = \rho_0$.

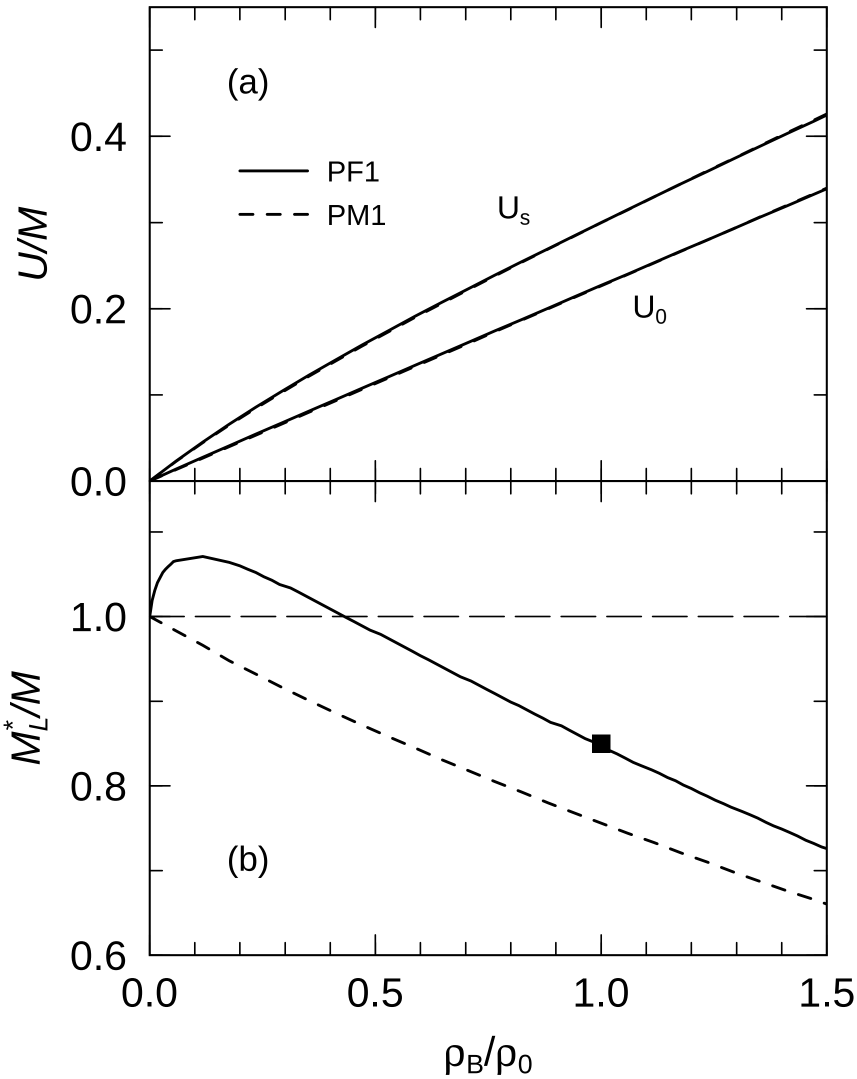

FIG. 2: Density-dependence of the Dirac selfenergies U_s and U_0 on the Fermi-surface (a) and the Landau mass (b) the Landau mass (b). The solid and dashed lines indicate the results for PF1 and PM1, respectively, and the full square in (b) denotes the value expected empirically from ISGQR.

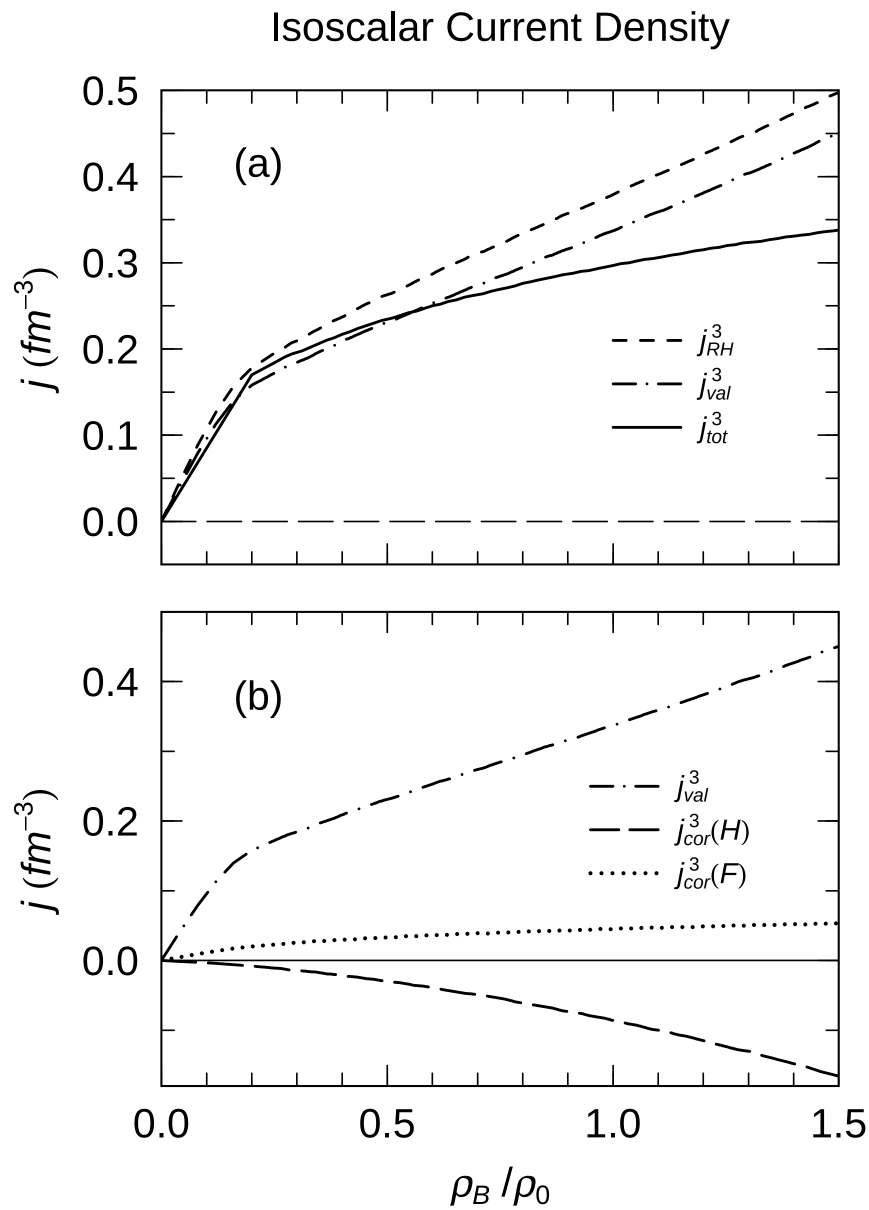

FIG. 3: Density-dependence of the isoscalar nuclear current density (a) and parts of the core polarization current density (b). The chain-dotted lines indicate the valence current densities. In the upper pannel (a) the dashed and solid lines represent the current density for the RH approximation and the total current densities, respectively. In the lower pannel (b) the dashed and dotted line represent the Hartree and Fock contributions to the core polarization current densities, respectively.

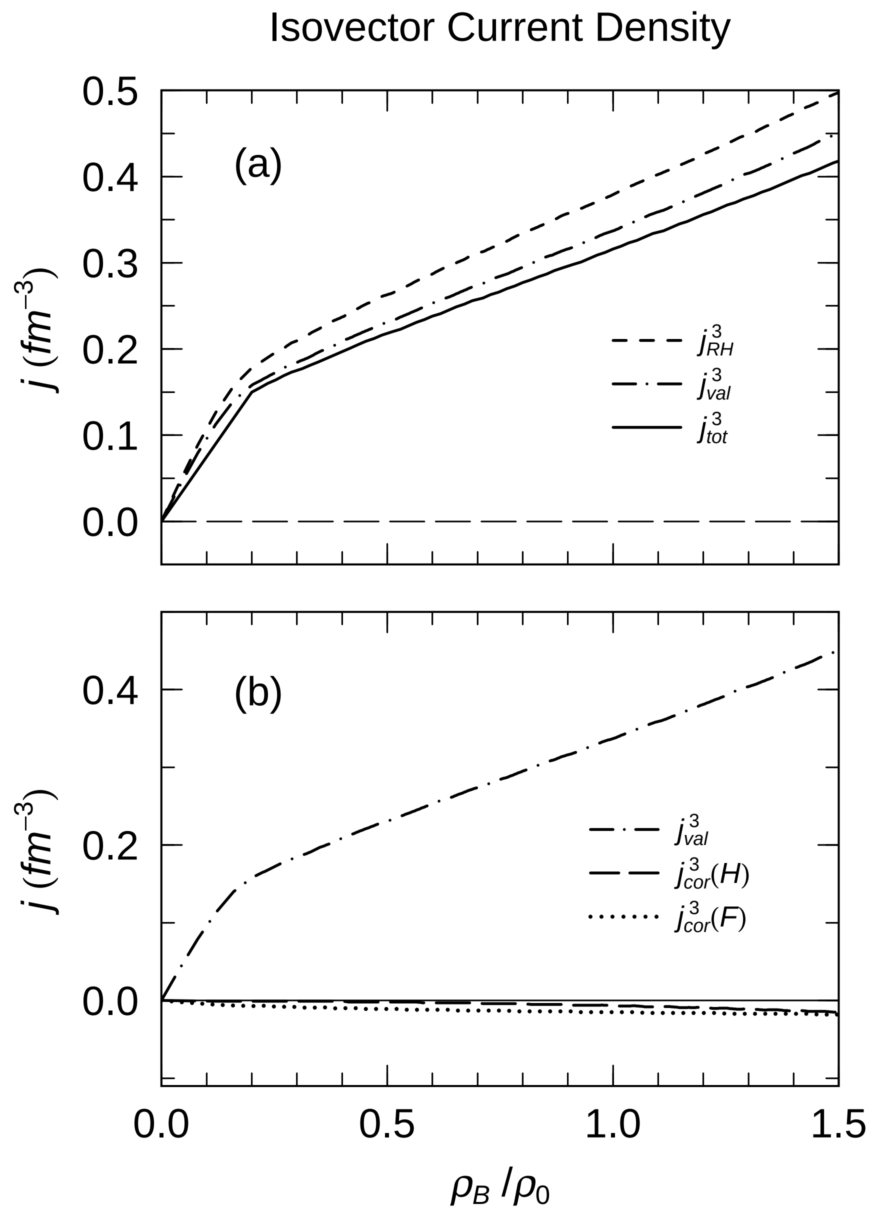

FIG. 4: Density-dependence of the isovector nuclear current density (a) and parts of the core polarization current density (b). The meanings of lines are the same as those in Fig. 3.

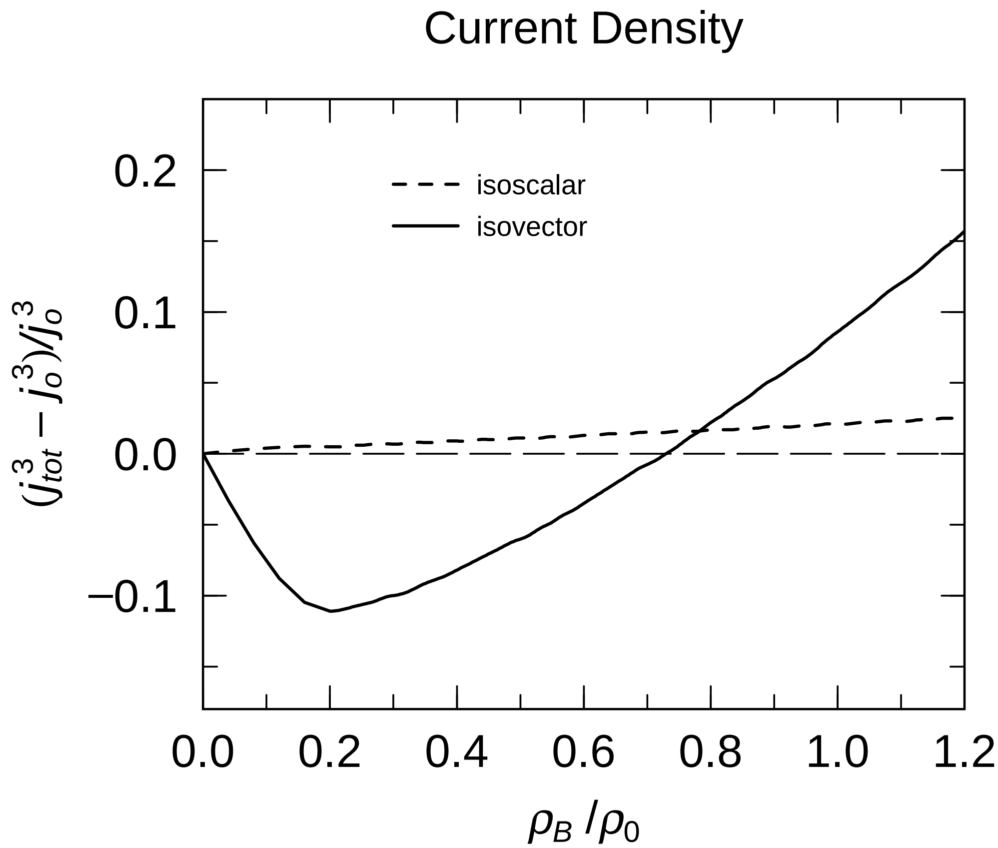

FIG. 5: The density dependence of the difference between the normal current density and the total current density in our model, normalized by the normal current density. The solid and dashed lines represent the isoscalar and isovector current densities, respectively.

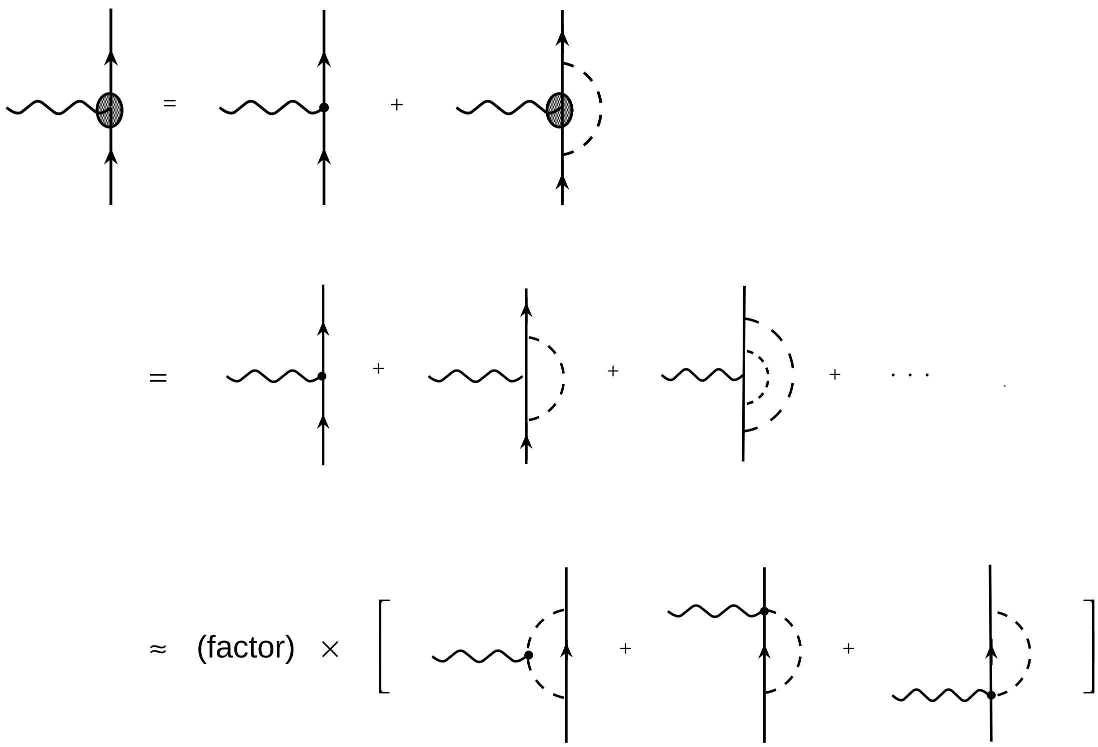

FIG. 6: Feynman diagram to show the vertex correction for the electro-magnetic current. The upper diagrams indicates the usual vertex correction, and the lower ones our approximate current. The solid, wave and dashed lines denotes the propagators of nucleon, meson and photon.